Neutrino physics using quantum coherent ions in circular motions

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Outline of this talk

Introduction

Idea: Use of quantum coherence for new source of neutrinos

Basic rate formulas and their features of neutrino pair beam
A few works in collaboration with N. Sasao

Neutrino oscillation and CPV parameter determination

R&D works using photon emission: see also Sasao at the poster session

Present status of neutrino physics

- Oscillation experiments
  - Finite mass
  - Flavor mixing
  - Only mass-squared difference can be measured.

$$U = VP\,, \tag{A8}$$

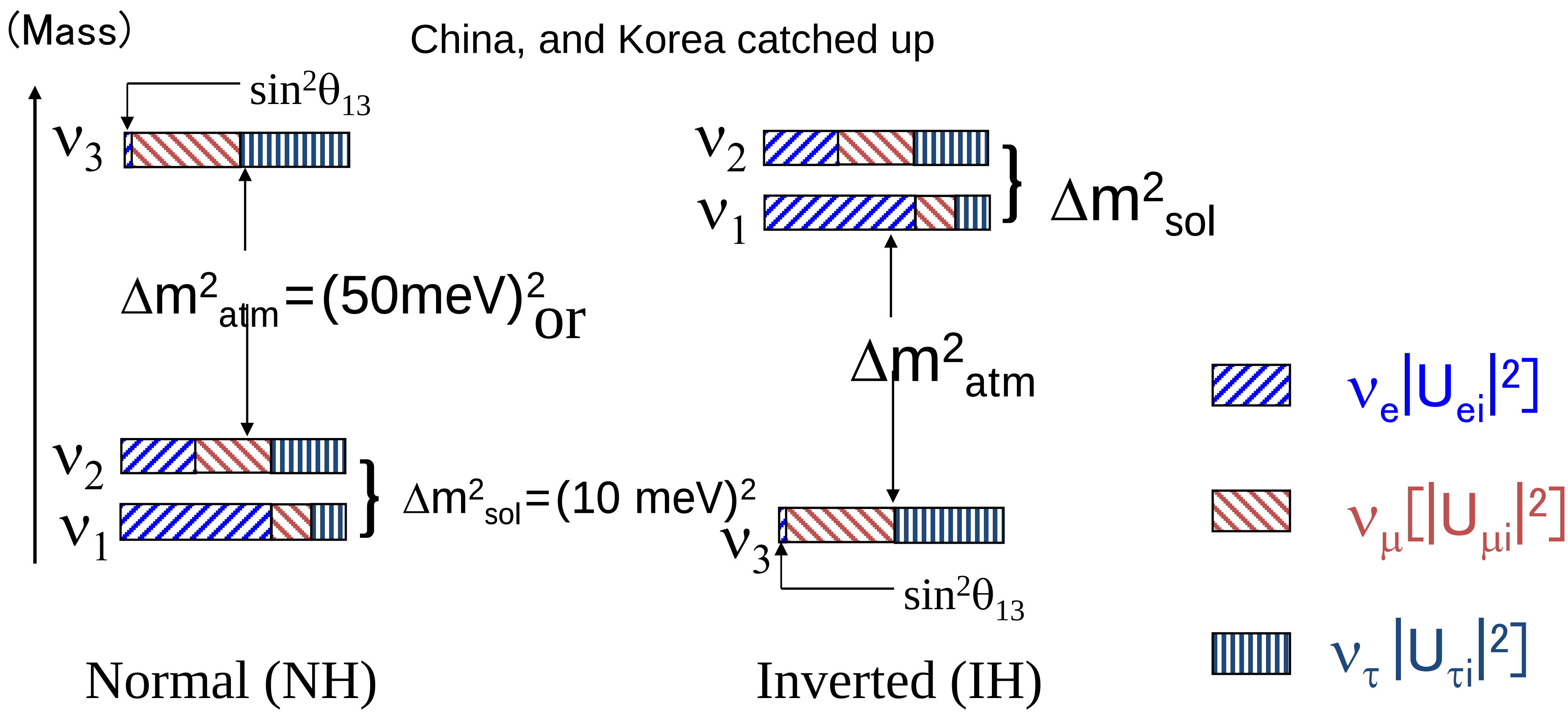
where

$$V = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix}, \tag{A9}$$

with $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$. The diagonal unitary matrix P may be expressed by

$$P = \mathrm{diag.}(1, e^{i\alpha}, e^{i\beta})\,, \tag{A10}$$

China, and Korea catched up



M. Yoshimura @RIKEN, 11/30/2015

Important questions left in neutrino physics

- Absolute mass scale and the smallest mass (oscillation experiments are sensitive to mass squared differences alone)
- Majorana vs Dirac distinction
- CPV phase （Majorana case has 2 extra phases）

$$\alpha\,,\ \beta\,,\ \delta\ (\mathrm{KM}-\mathrm{type})$$

These are relevant to explanation of matter-antimatter imbalance of universe and understanding the smallness of neutrino masses.

We wish to experimentally achieve some of these goals.

Conventional and new neutrino sources

- Decay product of elementary particles: pion, muon $\pi \to \mu\nu_\mu\,, \qquad \mu \to e + \nu_\mu \nu_e$
- Product of beta-decay $n \to p + e + \nu_e$

- Question: Is there another stronger source ?
- Answer: Yes, from ion de-excitation, but how about rate and spectrum calculated ?

Neutrino pair emission might occur similarly to synchrotron radiation,

But, producing neutrino pairs only in the keV energy region with extremely small rates, hence completely negligible for both electron synchrotron and circulating heavy ion in the ground state

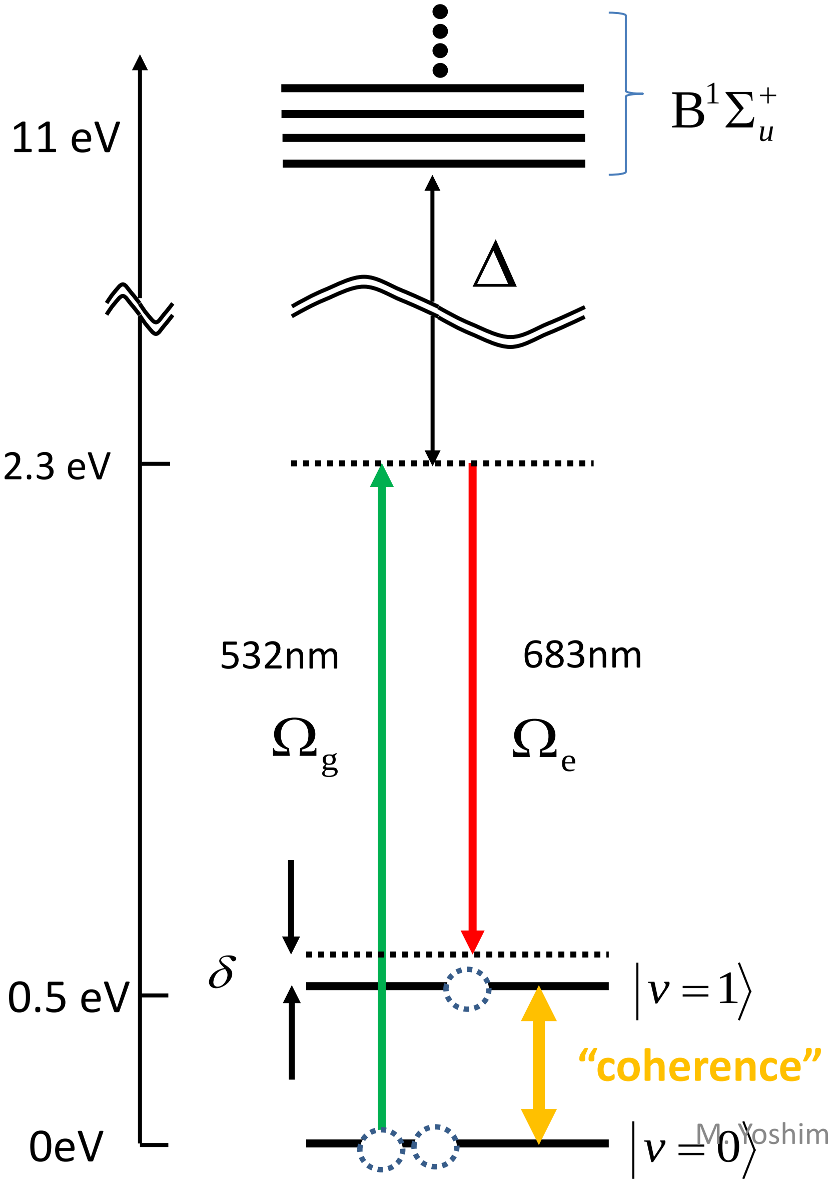
New feature from quantum circulating ions

Input of excitation energy ϵ_{eg} leading to a kind of non-linear resonance （positive and negative phase cancellation in a phase integral over times)

Quantum coherent state

what is it ?

Preparation of initial coherence – Adiabatic Raman -



Two laser fields irradiates p-H2

Two photon Rabi frequency $\Omega_{ge} \cong \dfrac{\Omega_g \Omega_e}{\Delta}$

→ |g> and |e> are mixed with an angle

$$\tan\theta \cong \frac{\Omega_{ge}}{\delta}$$

Non-degenerate Superposition States:

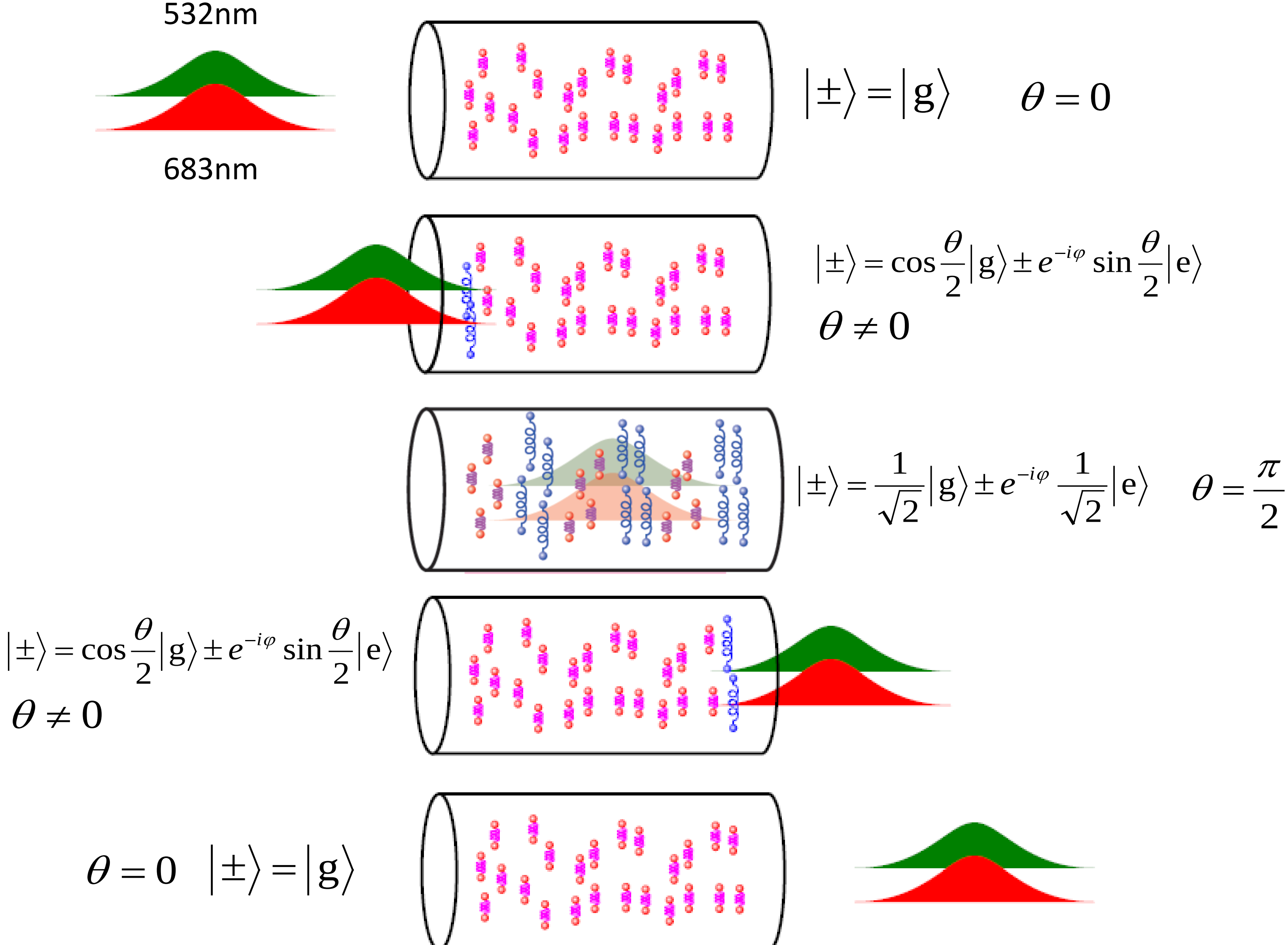
$$|+\rangle = \cos\frac{\theta}{2}|g\rangle + e^{-i\varphi}\sin\frac{\theta}{2}|e\rangle$$

$$|-\rangle = \cos\frac{\theta}{2}|g\rangle - e^{-i\varphi}\sin\frac{\theta}{2}|e\rangle$$

Coherence between |e> and |g>

$$\boxed{|\rho_{eg}| = \frac{1}{2}\sin\theta}$$

p-H2 gas



532nm
683nm
$|\pm\rangle = |\mathrm{g}\rangle \qquad \theta = 0$
$|\pm\rangle = \cos\frac{\theta}{2}|\mathrm{g}\rangle \pm e^{-i\varphi}\sin\frac{\theta}{2}|\mathrm{e}\rangle$
$\theta \neq 0$
$|\pm\rangle = \frac{1}{\sqrt{2}}|\mathrm{g}\rangle \pm e^{-i\varphi}\frac{1}{\sqrt{2}}|\mathrm{e}\rangle \qquad \theta = \frac{\pi}{2}$
$|\pm\rangle = \cos\frac{\theta}{2}|\mathrm{g}\rangle \pm e^{-i\varphi}\sin\frac{\theta}{2}|\mathrm{e}\rangle$
$\theta \neq 0$
$\theta = 0 \quad |\pm\rangle = |\mathrm{g}\rangle$

Some basic calculation of photon emission from quantum coherent ions in circular motion

$$|c(t)\rangle = \cos\theta_c e^{-i\epsilon_g t/\gamma}|g\rangle + \sin\theta_c e^{-i\epsilon_e t/\gamma} e^{i\varphi_c}|e\rangle$$

Quantum pure state of mixed $|e\rangle \;\&\; |g\rangle$ $\qquad \gamma = 1/\sqrt{1-\beta^2}$

$$\mathcal{M}(t) = \int_0^t dt' \langle c(t')|\frac{e\vec{p}}{m_e}\cdot\vec{A}(t';\vec{k},h)|c(t')\rangle$$
$$= -i\frac{\epsilon_{eg}}{\sqrt{2\omega V}}\int_0^t dt' \langle c(t')|\vec{d}|c(t')\rangle \cdot \vec{e}_h e^{i\omega t' - i\vec{k}\cdot\vec{r}_I(t')}$$

Expectation value instead of transition amplitude

$\frac{p\cdot A}{m_e}$ gauge instead of $d\cdot E$

$$\vec{r}_I(t) = \rho(\sin\frac{vt}{\rho}, \cos\frac{vt}{\rho}, 0) \quad \vec{k} = \omega(\cos\psi\cos\theta, \cos\psi\sin\theta, \sin\psi)$$

Single ions contributing many, many times to emission of the same photon

$$d\Gamma = \frac{V d^3 k}{(2\pi)^3}\frac{2\pi\rho I}{Q}\gamma \sum_h 2\Re\left(\langle c(t)| \frac{e\vec{p}}{m_e}\cdot \vec{A}(t;\vec{k},h)|c(t)\rangle \mathcal{M}(t)^* \right) ,$$

$$2V\Re\left(\langle c(t)| \frac{e\vec{p}}{m_e}\cdot \vec{A}(t;\vec{k},h)|c(t)\rangle \mathcal{M}(t)^* \right)$$

$$= \frac{\epsilon_{eg}^2}{\omega}(\sin\theta_c\cos\theta_c)^2 \sum_{pol} e_h^i (e_h^j)^*(d_{eg})_i (d_{eg})_j \cos\tilde{\Phi}(0)\int_0^t dt'\cos\tilde{\Phi}(t') ,$$

$$\tilde{\Phi}(t) = (\omega - \frac{\epsilon_{eg}}{\gamma})t - \rho\omega\cos\psi\sin(\theta + \frac{\beta t}{\rho}) .$$

Absent in synchrotron radiation formula

Differential rates for photon emission

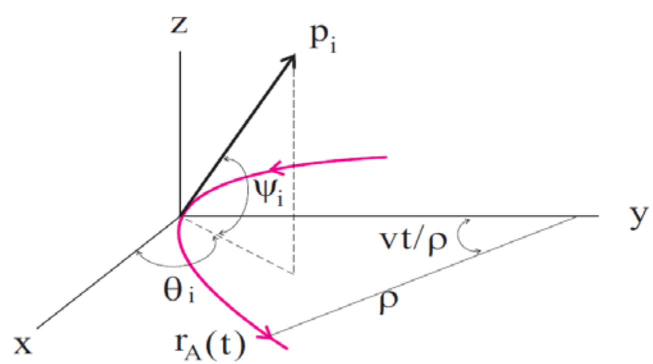
$$\frac{d^2\Gamma}{d\omega d\Omega} = \frac{8}{3(2\pi)^2} N d_{eg}^2 \omega^3 \gamma \frac{\rho}{\beta} \int_0^{\beta t/\rho} du \cos \Phi(u) \,, \qquad N = |\rho_{eg}(t)|^2 \frac{\rho I}{Q}$$

$$\Phi(u) = \frac{\rho}{\beta}(\omega - \frac{\epsilon_{eg}}{\gamma})u - \rho\omega \cos\psi \cos\theta \sin u - \rho\omega \cos\psi (\cos u - 1)$$

$$\sim \frac{\rho}{\beta}(\omega - \frac{\epsilon_{eg}}{\gamma})u - \rho\omega \cos\psi \cos\theta \sin u \equiv bu - u \sin u \,,$$

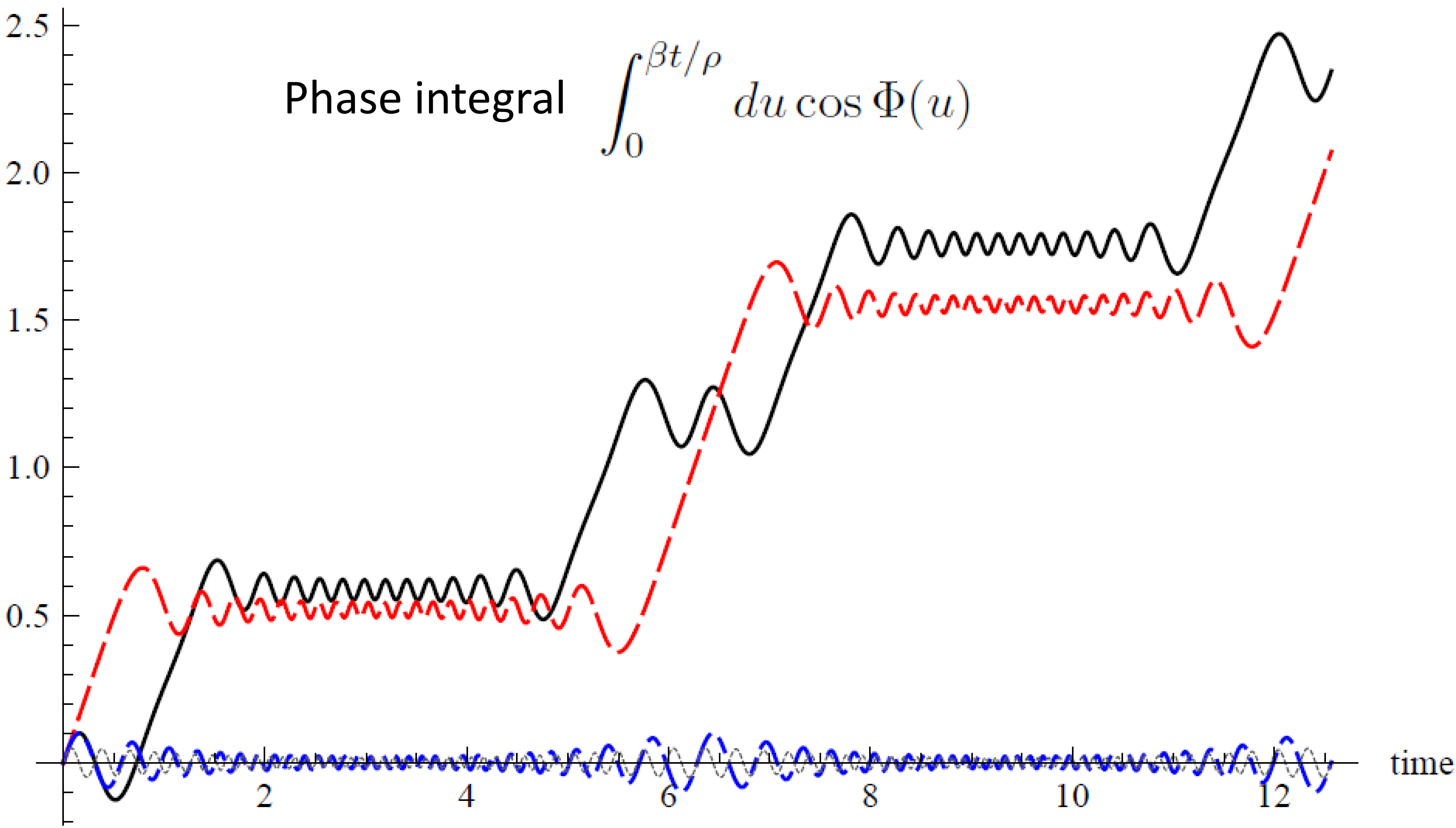
$$b = \frac{\rho}{\beta}(\omega - \frac{\epsilon_{eg}}{\gamma}) \,, \qquad c = \rho\omega \cos\psi \cos\theta \,.$$

$$\rho\epsilon_{eg} \sim 2.5 \times 10^{10} \frac{\rho}{1\mathrm{km}} \frac{\epsilon_{eg}}{5\mathrm{eV}}$$

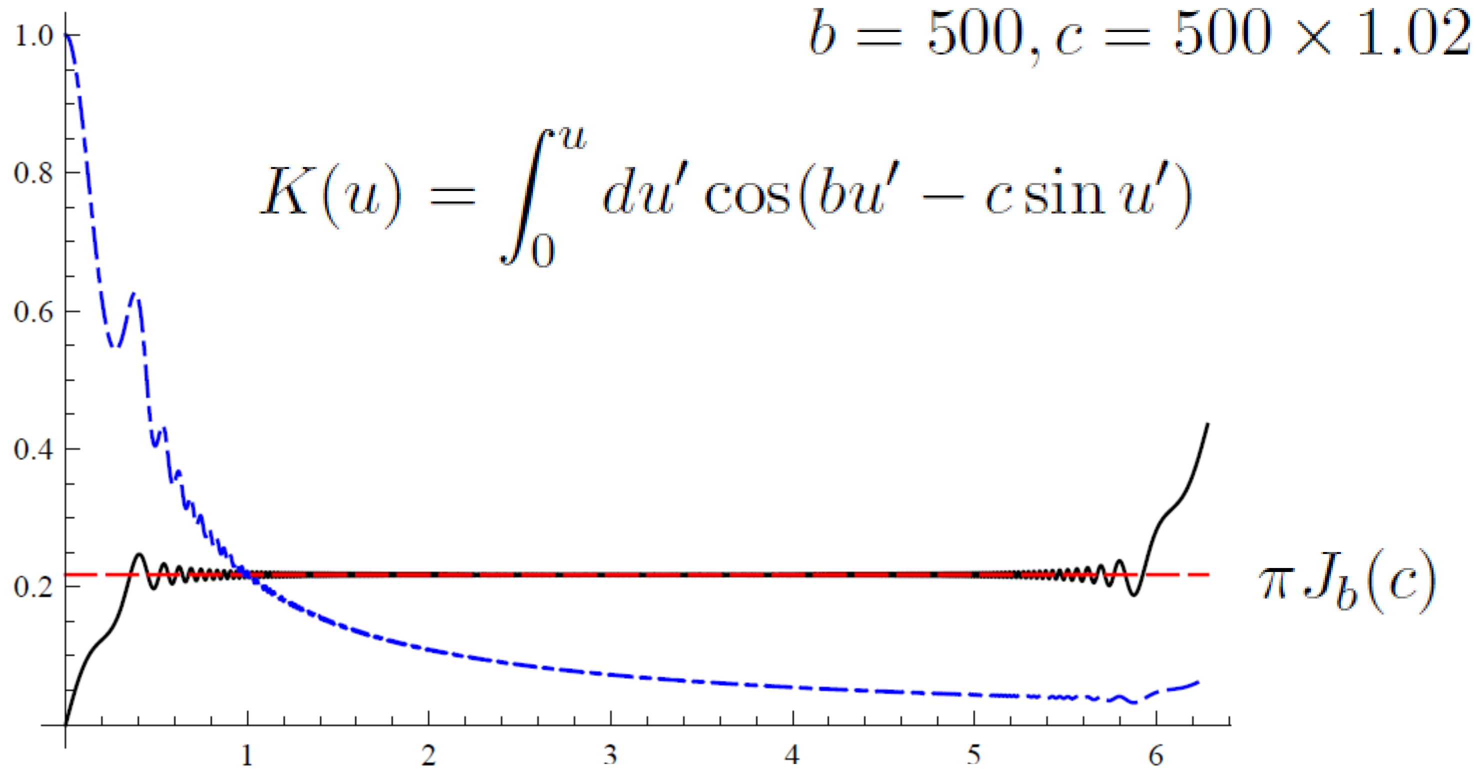


$100\,\mu$radian $10^4/\gamma$

Rate functions



Large phase integral -> Bessel function of large order and large argument



- Useful formula

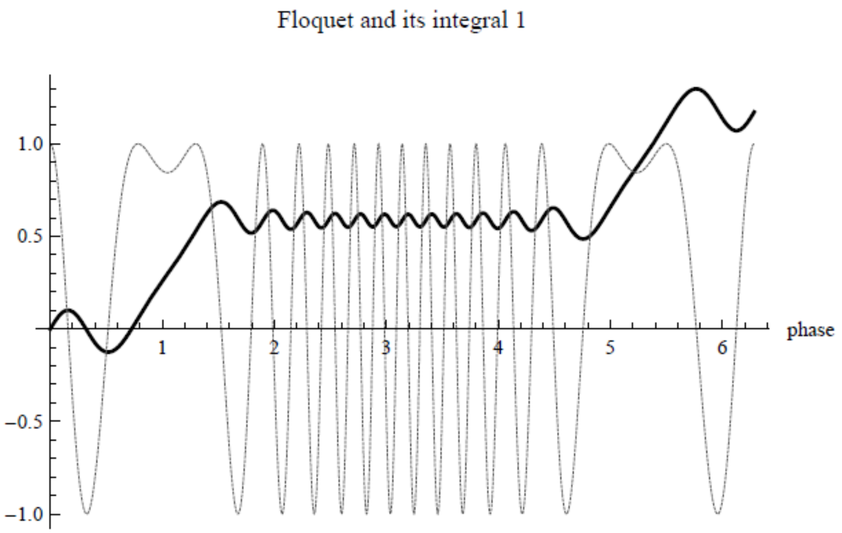
$$J_b(c) \sim \frac{1}{\sqrt{\pi}}(c^2 - b^2)^{-1/4}$$

$$0 < c^2 - b^2 \ll |b| \to \infty$$

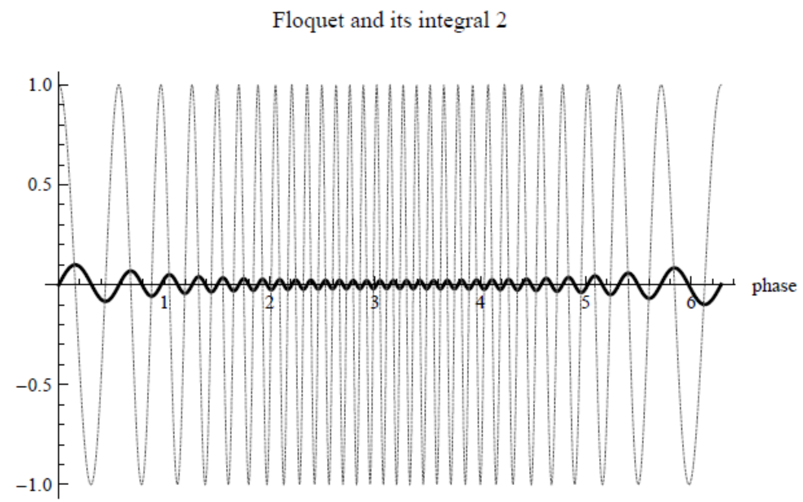
b, c of order $\rho\epsilon_{eg} \sim 2.5 \times 10^{10} \frac{\rho}{1\mathrm{km}} \frac{\epsilon_{eg}}{5\mathrm{eV}}$

Two cases of large production and failure

Phase function and its integral are plotted



Steady increase with one period of cicrulation



Failed case

Related to Floquet system of stability/instability band structure

Intuitive understanding

- A kind of non-linear resonance: orbital energy balanced against internal ion energy, giving non-linear resonance oscillation. Its width around the stationary point gives a narrow resonance-like behavior in time domain.

- Key concept for its success: quantum coherence typically realized by ionic system under laser irradiation, but may persist without phase relaxation.

- Macro-coherence not required

Simple example of quantum coherence: adiabatic Raman process

Difference from usual synchrotron radiation

using a truncated time expansion

Ground state ion

X = rescaled time

$$\int_0^\infty dx h(x) \cos \xi (\frac{1}{2}x^3 + \frac{3}{2}x) \to \sqrt{\frac{\pi}{6}} e^{-\xi} \frac{h(0)}{\sqrt{\xi}} .$$

$$\xi = \rho(E_1 + E_2) \times \text{ a function of } \left(\frac{E_1}{E_2}, \frac{\epsilon_{eg}}{E_1 + E_2}, \gamma, \text{angles} \right)$$

- Always the same sign phase added, leading to exponential dumping

Excited ion with coherence

$$\int_0^\infty dx h(x) \cos \xi (\frac{1}{2}x^3 - \frac{3}{2}x) \to \sqrt{\frac{2\pi}{3}} \cos(\xi - \frac{\pi}{4}) \frac{h(1)}{\sqrt{\xi}}$$

Cancellation of positive and negative phases

Energy input leads to resonance-like behavior

Result 1: Angular distribution

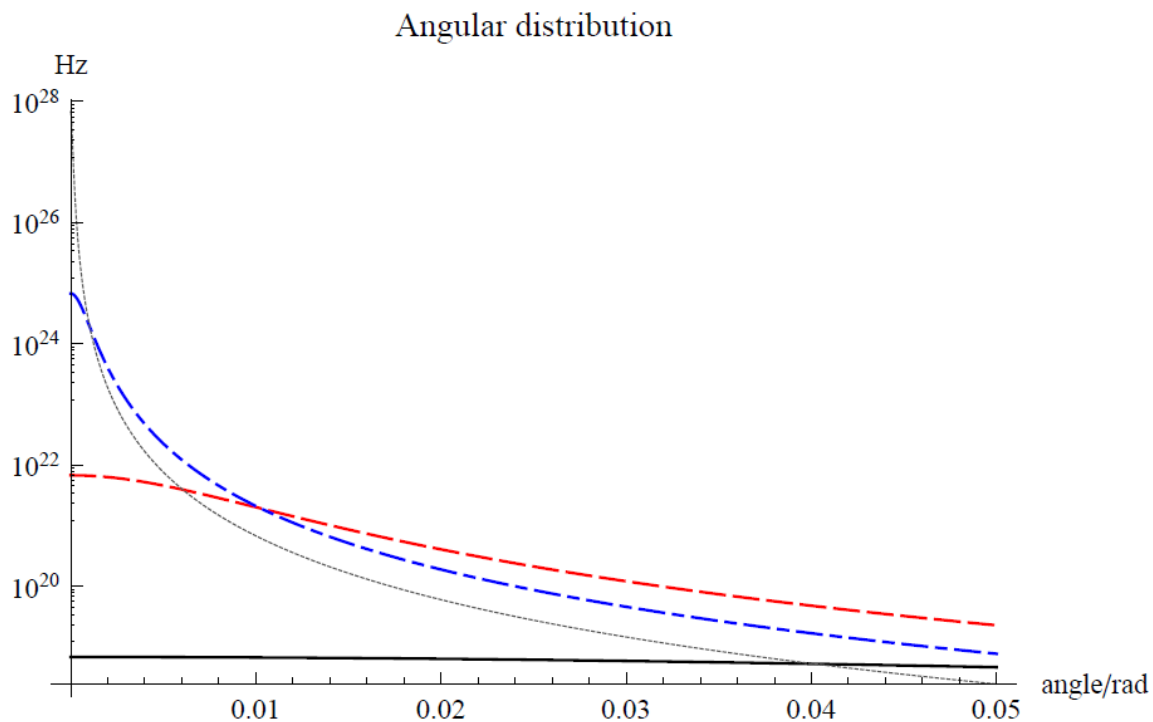


Figure 3: Angular distribution either at $\psi = 0$ or $\theta = 0$ in the single photon emission for a few choices of the boost factor: $\gamma = 10$ in solid black, 100 in dashed red, 1000 in dash-dotted blue, and 10^4 in dotted black. Other assumed parameters are A_{eg} =1kHz, $\rho\epsilon_{eg} = 10^{10}, N = 10^8$ and rates scale as $\propto A_{eg}\sqrt{\rho\epsilon_{eg}}N$.

Result 2: Photon energy spectrum at the forward

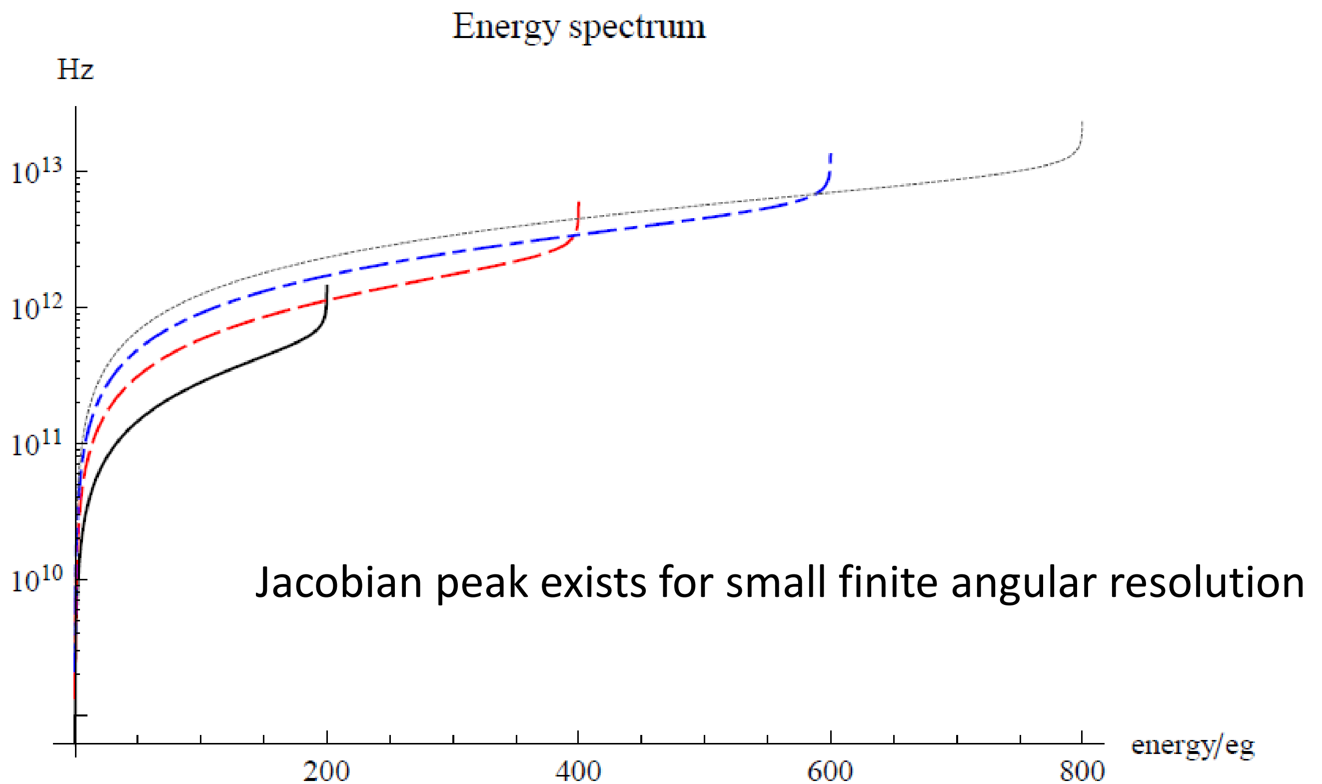
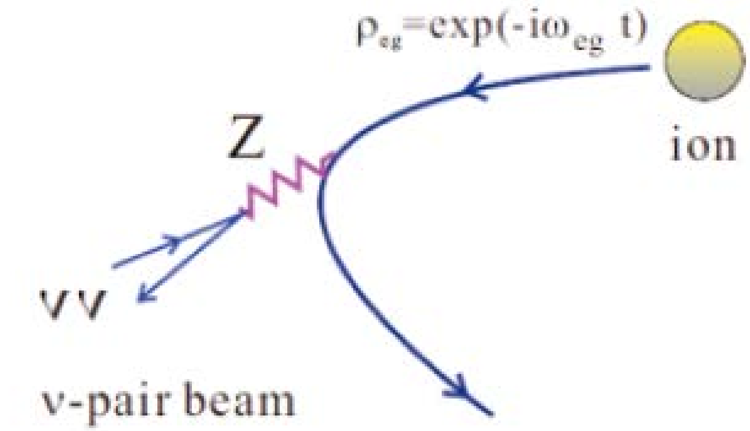


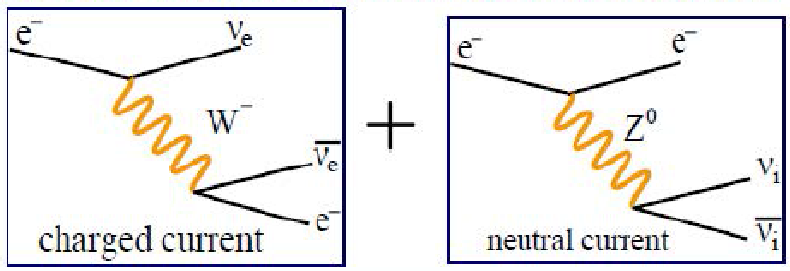
Figure 2: Photon energy spectrum per unit solid angle area at the forward direction for a few choices of the boost factor: $\gamma = 100$ in solid black, 200 in dashed red, 300 in dash-dotted blue, and 400 in dotted black. The angular resolution of $\Delta = 0.01/\gamma$ was taken here. Other assumed parameters are A_{eg} =1kHz, $\rho\epsilon_{eg} = 10^{10}, N = 10^8$ and rates scale as $\propto A_{eg}\sqrt{\rho\epsilon_{eg}}N$.

Case of neutrino pair emission



W-exchange, too

Neutrino interaction with atomic electron



4 Fermi interaction (after Fierz transformation)

Creation of atomic e

Annihilation of atomic e

$$\mathcal{H}_W = \frac{G_F}{\sqrt{2}}\bar{\nu}^e\gamma_\alpha(1-\gamma_5)\nu^e\bar{e}\gamma^\alpha(1-\gamma_5)e$$
$$-\frac{G_F}{2\sqrt{2}}\sum_i \bar{\nu}^i\gamma_\alpha(1-\gamma_5)\nu^i\bar{e}(\gamma^\alpha(1-4\sin^2\theta_W-\gamma_5))e$$

where $\nu_e = \sum_i U_{ei}\nu_i$ $\quad U_{\alpha i} =$ MNS matrix elements

- Completely coherent CP-even pairs of neutrinos in all flavors are produced if level spacing and boost factor are both large

Structure of weak current

- A and V contributions in electron-neutrino interaction
- A term:

$$\frac{G_F}{2\sqrt{2}}\bar{e}\gamma_\alpha\gamma_5 e\sum_b c_b^A\bar{\nu}_b\gamma^\alpha(1-\gamma_5)\nu_b$$

$$(|c_b^A|^2) = (1,1,1)/4 \propto 1$$

- V term:

$$\frac{G_F}{2\sqrt{2}}\bar{e}\gamma_\alpha e\sum_b c_b^V\bar{\nu}_b\gamma^\alpha(1-\gamma_5)\nu_b$$

$$(c_b^V) = (\frac{1}{2}(1+4\sin^2\theta_w)\,, -\frac{1}{2}(1-4\sin^2\theta_w)\,, -\frac{1}{2}(1-4\sin^2\theta_w))$$

Collimated neutrino beam

- Beam dump stronger photons in extraction system
- If coherence is not lost in the initial phase, neutrino experiments are possible
- CP-even neutrino pairs of all flavors are available
- LHC at CERN achieved acceleration of Pb^{82+}

Single neutrino energy spectrum at the forward direction

$$\frac{d\Gamma}{dy} = R\frac{\gamma^{15/2}}{\sqrt{\beta}}(1+\beta)^6 G(y)\,, \qquad y = \frac{E}{\epsilon_{eg}}\sqrt{\frac{1-\beta}{1+\beta}}\,,$$

$$R = \frac{1}{128\cdot 4\sqrt{\pi}(2\pi)^5} G_F^2 \epsilon_{eg}^5 \frac{A_{eg}}{\alpha\epsilon_{eg}} N\sqrt{\rho\epsilon_{eg}} \sim 2.1\times 10^{-17}\mathrm{Hz}\frac{A_{eg}}{\mathrm{kHz}}\left(\frac{\epsilon_{eg}}{10\mathrm{keV}}\right)^4 \frac{N}{10^8}\sqrt{\frac{\rho\epsilon_{eg}}{10^{14}}}\,,$$

$$G(y) = y^2\int_{y_- - y}^{1-y} dy_2 y_2^2\left((y+y_2-y_-)(1-y-y_2)\right)^{-1/4}\,, \qquad y_- = \frac{1-\beta}{1+\beta}\,.$$

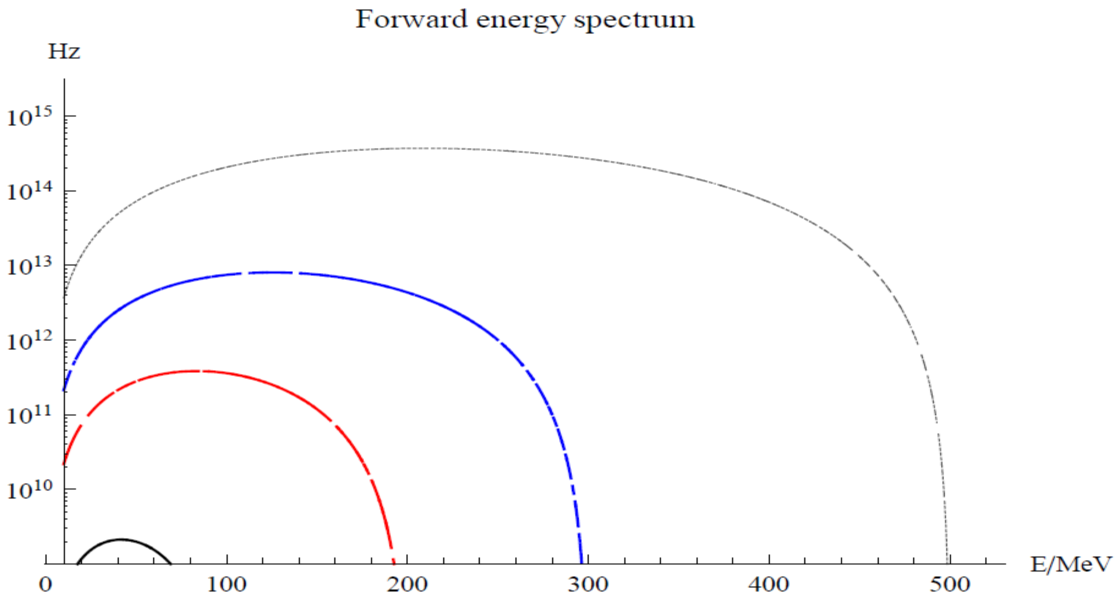


Figure 5: Neutrino energy spectrum rate at the forward direction per unit solid angle area. Assumed parameters are ρ =5km, ϵ_{eg} =50 keV, γ = 1000 in solid black, 2000 in dashed red, 3000 in dash-dotted blue, and 5000 in dotted black.

Neutrino oscillation

Work in progress with T. Asaka and M. Tanaka

Disappearance in A and appearance in V of oscillation patterns

- A coherent pair of neutrinos propagates at distance L, and suppose we detect one of the pair

$$\langle c|e^{-iHL}|b\rangle = \sum_i V_{ci}^* V_{bi} e^{-i\lambda_i L} \,, \qquad \langle \bar{a}|e^{-iHL}|\bar{b}\rangle = \sum_i \bar{V}_{ai}^* \bar{V}_{bi} e^{-i\bar{\lambda}_i L} \,,$$

$$\sum_b \langle \bar{a}|e^{-iHL}|\bar{b}\rangle \langle c|e^{-iHL}|b\rangle c_b = \sum_{ij} V_{ci}^* \bar{V}_{aj}^* \xi_{ij} e(\bar{\lambda}_j, \lambda_i) \,,$$

$$\xi_{ij} = c_e \bar{V}_{ej} V_{ei} + c_\mu \bar{V}_{\mu j} V_{\mu i} + c_\tau \bar{V}_{\tau j} V_{\tau i} \,, \qquad e(\bar{\lambda}_j, \lambda_i) = \exp[-iL(\lambda_i + \bar{\lambda}_j)] \,.$$

$$\sum_c |\sum_{ij} V_{ci}^* \bar{V}_{aj}^* \xi_{ij} e(\bar{\lambda}_j, \lambda_i)|^2 = \sum_{ijkl} \sum_c V_{ci}^* V_{ck} \bar{V}_{aj} \bar{V}_{al}^* \xi_{ij} \xi_{kl}^* e(\bar{\lambda}_j, \lambda_i) e^*(\bar{\lambda}_l, \lambda_k)$$

$$= \sum_{jl} \bar{V}_{aj} \bar{V}_{al}^* e(\bar{\lambda}_j, \lambda_i) e^*(\bar{\lambda}_l, \lambda_i) \sum_i \xi_{ij} \xi_{il}^* \,.$$

V:

$$P_a(E,L;m_i,\delta) \equiv \sum_{jl} p_{jl} \bar{V}_{aj}{}^{*} \bar{V}_{al} \exp[-i(\bar{\lambda}_j - \bar{\lambda}_l)L]$$

$$= \frac{1}{4}(1-4\sin^2\theta_w)^2 + 4\sin^2\theta_w \sum_{jl} \bar{V}_{ej}\bar{V}_{el}^{*}\bar{V}_{aj}^{*}\bar{V}_{al} \exp[-i(\bar{\lambda}_j - \bar{\lambda}_l)L]$$

$$\sim 0.00058 + 0.952 \sum_{jl} U_{ej}U_{el}^{*}U_{aj}^{*}U_{al} \exp[-i\delta m_{jl}^2 \frac{L}{2E}]\,,$$

$$\sum_{jl} U_{ej}U_{el}^{*}U_{aj}^{*}U_{al} \exp[-i\delta m_{jl}^2 \frac{L}{2E}] = |\sum_{j} U_{ej}U_{aj}^{*} \exp[-i\frac{m_j^2 L}{2E}]|^2\,,$$

A: $$|c^A|^2 \sum_{jl} \bar{V}_{aj}^{*}\bar{V}_{al} e(\bar{\lambda}_j, \lambda_i) e^{*}(\bar{\lambda}_l, \lambda_i)\delta_{jl} = |c^A|^2 = \frac{1}{4}$$

Basic oscillation patterns

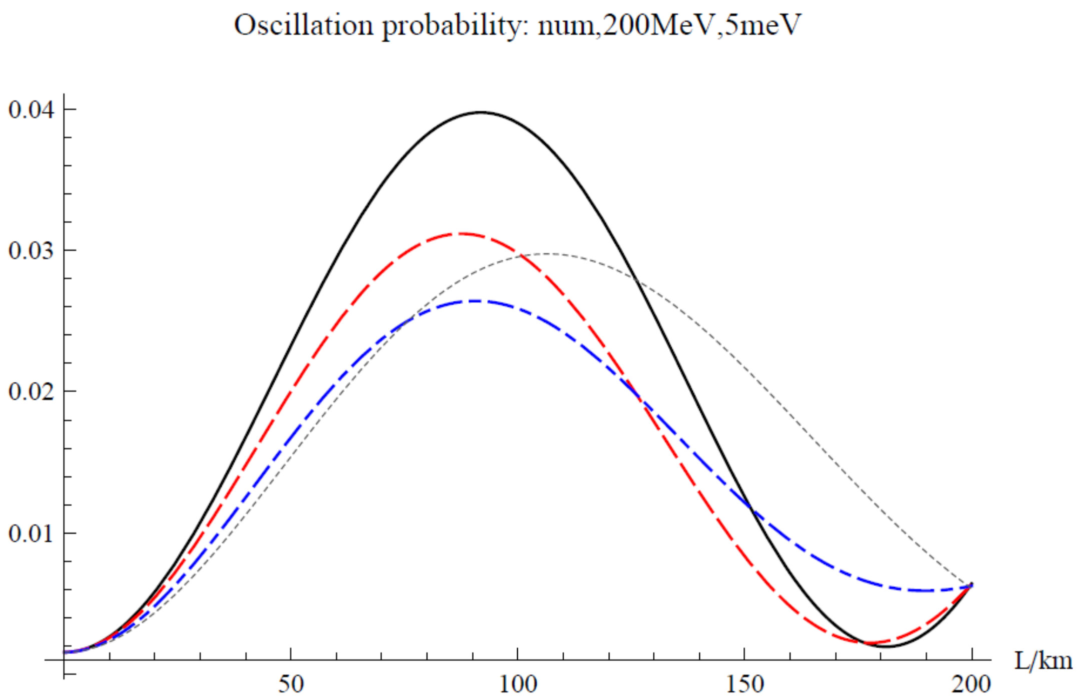
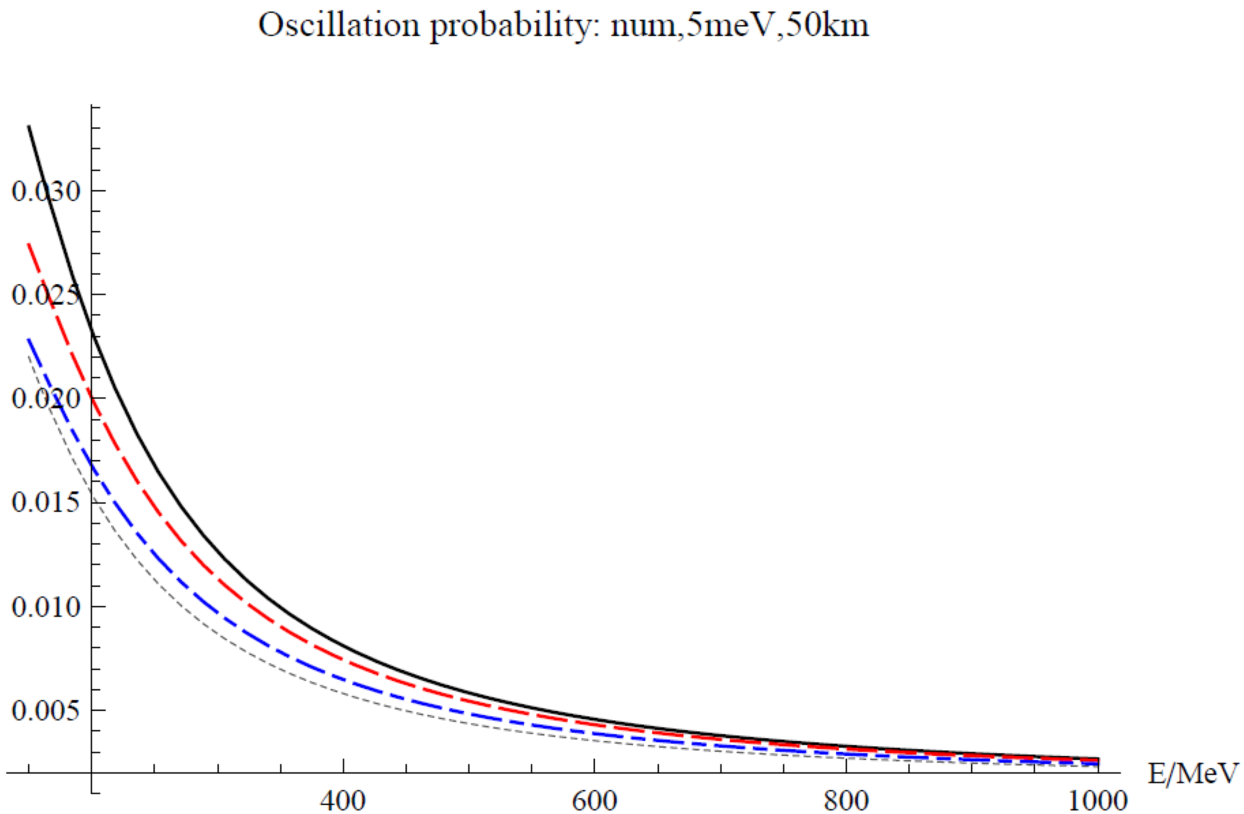


Figure 1: ν_μ survival probability using eq.(18) for a few choices of CPV parameter δ: 0 in solid black, $\pi/4$ in dashed red, $\pi/2$ in dash-dotted blue, and $3\pi/4$ in dotted black. Other assumed parameters are the smallest neutrino mass 5meV in NH, the neutrino energy fixed at 200 MeV.

Short baseline oscillation experiments possible within 1/g collimated direction and detector placed on earth

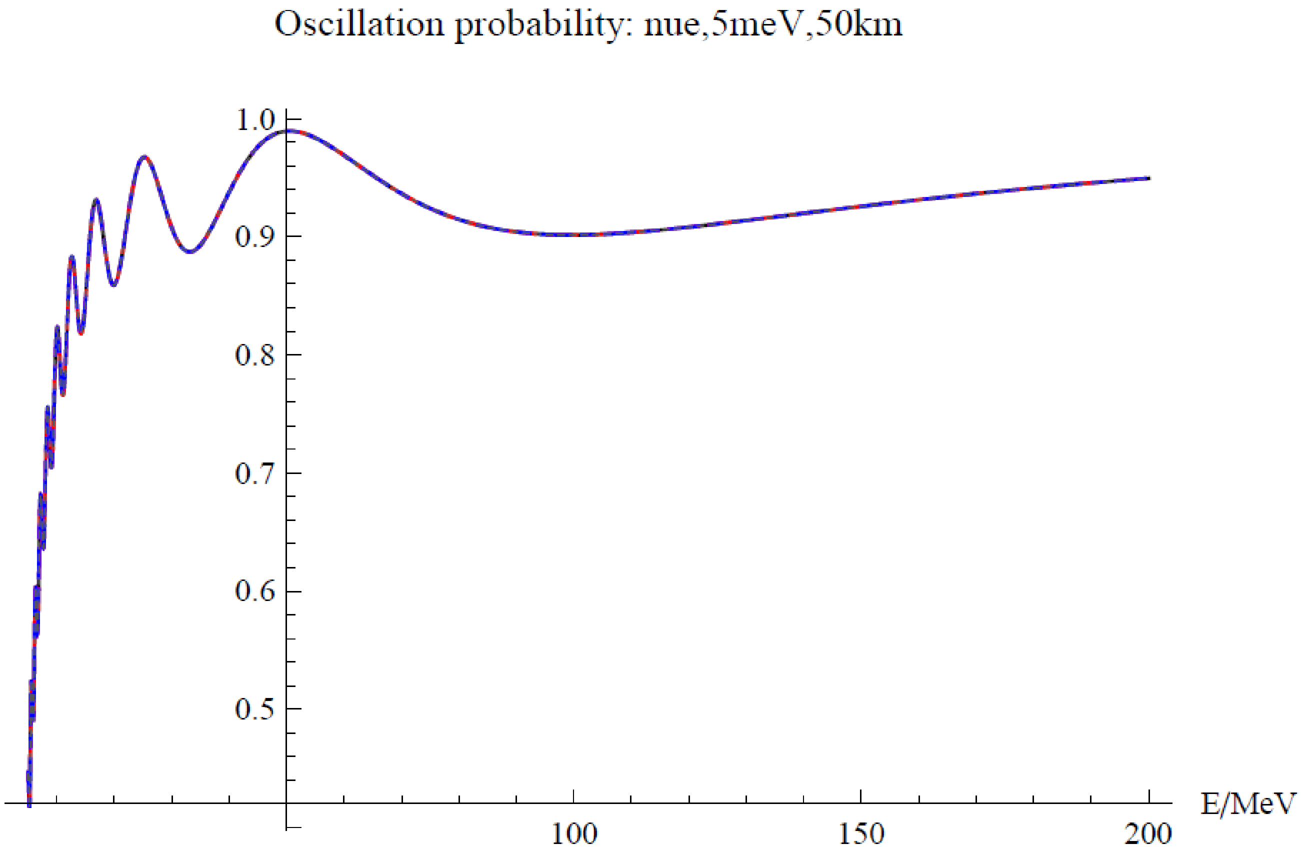
Differences for different CPV parameters



listinguished

Figure 2: ν_μ survival probability using eq.(18) for a few choices of CPV parameter δ: 0 in solid black, $\pi/4$ in dashed red, $\pi/2$ in dash-dotted blue, and $3\pi/4$ in dotted black. Assumed parameters are the smallest neutrino mass 5meV in NH, the distance 50 km away from the ring.

Nu_e detection



No chance of CPV parameter determination

Search for the best candidate ion for circulation needs help

- E1-like, deep levels for large spacing, and hopefully smaller A are required

Be-like; 2s2p $^3P_1^-$ of harge Z−4 $\rightarrow ^1S_0^+$
Ne-like 2p$^+$ 3s $^3P_1^-$ of charge Z−10 $\rightarrow ^1S_0^+$
Both having competition of A-contribution from $\rightarrow ^3P_0^-, ^3P_2^-$?

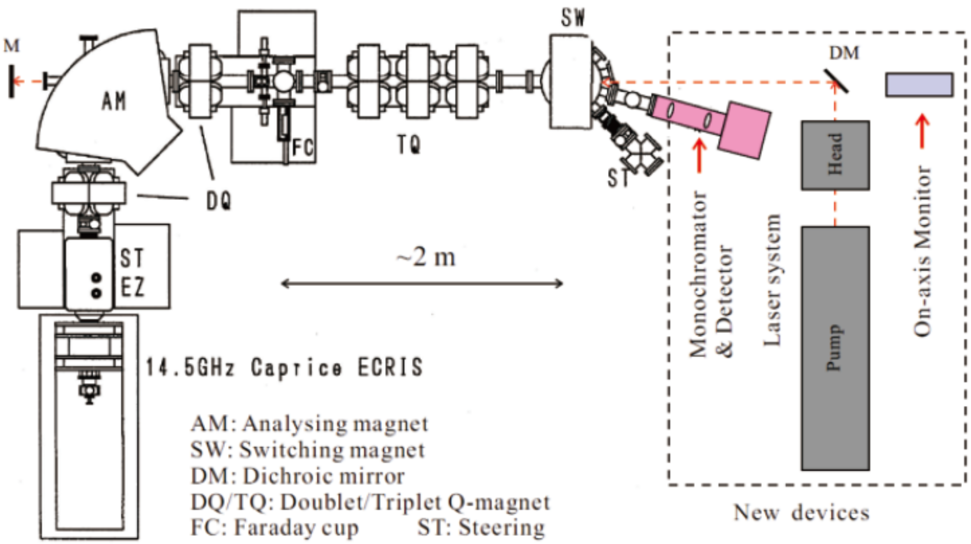
Need relativistic atomic physics calculation of ϵ_{eg} , A_{eg}

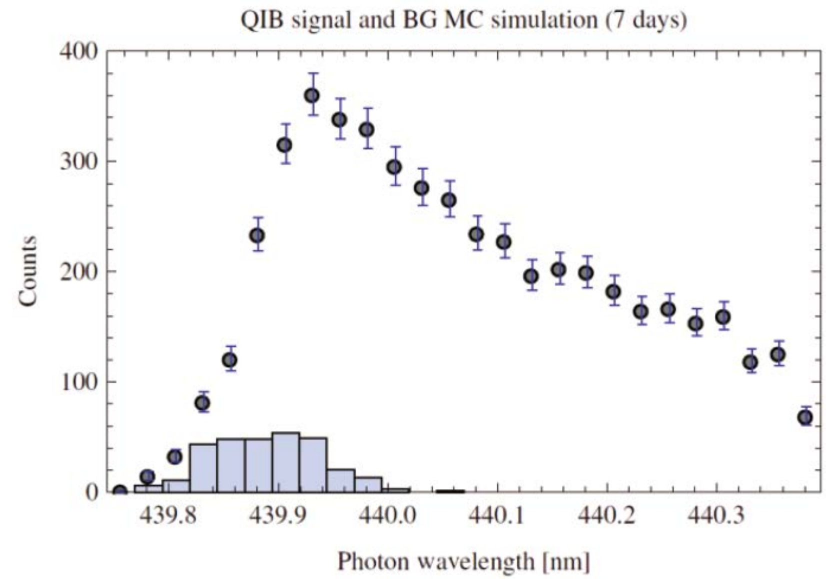
Coherent gamma ray beam

- If the macro-coherence is achieved within a bunch of ions, the coherent gamma emission is expected, like the laser in the optical region.

- This would lead to the possibility of very fast and fine resolved imaging of atoms, nucleus and atomic electron cloud separately snap-shoted !

R&D works using low energy ion facility of RIKEN





- See Sasao at the poster session

Summary of this talk

- Coherent quantum heavy ion synchrotron is excellent for CPV parameter, NH/IH hierarchy determination (pair beam).

- Accelerator R & D works using photon emission is crucial to learn how to achieve a high coherence ion.

- Bonus from R&D may lead to unexpected “gamma ray laser”